TYPE 560: COMBINED PHOTOVOLTAIC / THERMAL SOLAR COLLECTOR (INTERACTS WITH ZONE AIR TEMPERATURE)

This component is intended to model an un-glazed solar collector which has the dual purpose of creating power from embedded photovoltaic (PV) cells and providing heat to a fluid stream passing through tubes bonded to an absorber plate located beneath the PV cells. The waste heat rejected to the fluid stream is useful for two reasons; 1) it cools the PV cells allowing higher power conversion efficiencies and 2) it provides a source of heat for many possible low-grade temperature applications.

This model relies on linear factors relating the efficiency of the PV cells to the cell temperature and also the incident solar radiation. The cells are assumed to be operating at their maximum power point condition.

The thermal model of this collector relies on algorithms presented in Chapter 6 of the classic "Solar Engineering of Thermal Processes" textbook by Duffie and Beckman.

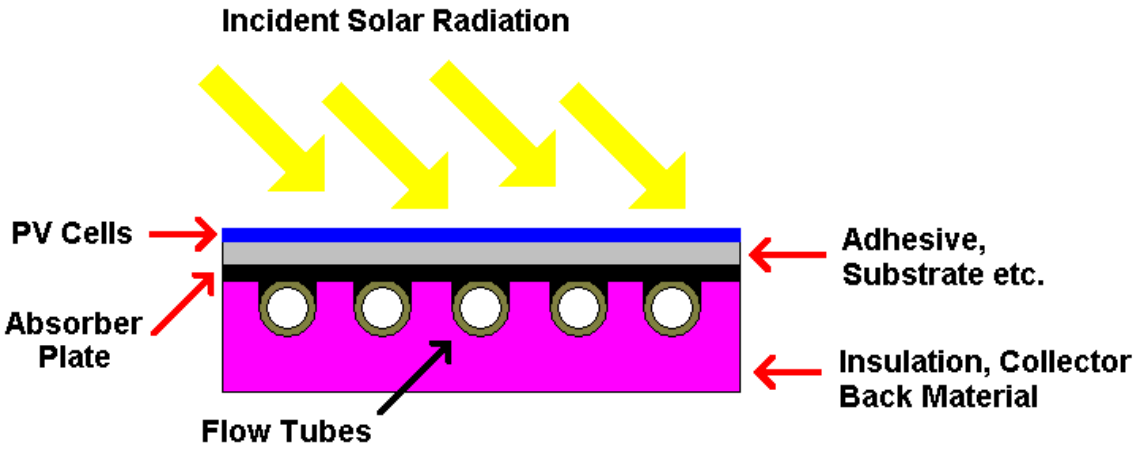


Figure 1 –PV/T Schematic

Nomenclature

β	- slope of the collector surface
η	- efficiency
θ	- angle of incidence
ρ	- ground reflectance
$\tau\alpha$	- transmittance-absorptance product for the solar collector
ε	- emissivity of the top surface of the collector (PV surface)
σ	- Stefan-Boltzmann constant
λ	- thickness of the absorber plate

Area	- area (top) of the solar collector; this can be either gross area or net area but should be consistent with the provided loss coefficients and PV power conversion coefficients.
b_0	- incidence angle modifier multiplier
C_p	- specific heat of the fluid flowing through the PV/T collector
C_B	- the conductance between the absorber plate and the bonded tube
D_{tube}	- the diameter of the tubes
F_R	- collector heat removal factor
G_t	- total solar radiation (beam + diffuse) incident upon the collector surface
h_{fluid}	- internal fluid heat transfer coefficient
h_{inner}	- heat transfer coefficient from the back of the collector to the air
h_{outer}	- heat transfer coefficient from the top of the collector (PV surface) to the ambient air
h_{rad}	- radiative heat transfer coefficient from the top of the collector (PV surface) to the sky
IAM	- incidence angle modifier
k	- thermal conductivity of the plate material
L	- the length of the collector along the flow direction
$\dot{m}$	- flow rate of fluid through the solar collector
N_{tubes}	- number of identical tubes carrying fluid through the collector
Power	- rate at which electrical energy is produced by the PV cells
$Q_{loss,top,conv}$	- rate at which energy is lost to the ambient through convection off the top of the collector
$Q_{loss,top,rad}$	- rate at which energy is lost to the sky through radiation off the top of the collector
$Q_{loss,back}$	- rate at which energy is lost to the ambient through the back of the collector
Q_{fluid}	- rate at which energy is added to the flow stream by the collector, this term includes the energy that is also lost from the fluid stream through the back of the collector
$Q_{absorbed}$	- net rate at which energy is absorbed by the collector plate (does not include PV power production)
Q_u	- rate at which energy is added to the flow stream by the collector
q'_{fin}	- heat transfer to the fin base per unit length of collector
q'_{fluid}	- heat transfer to the fluid stream per unit length of collector
q'_u	- heat transfer to the fluid stream per unit length of collector
R_t	- resistance to heat transfer from the PV cells to the absorber plate
R_b	- resistance to heat transfer from the absorber through the back of the collector
R_1	- resistance to heat transfer provided by the material between the PV cells and the absorber
R_2	- resistance to heat transfer provided by the material between the absorber plate and the back surface of the collector
S	- net absorbed solar radiation (total absorbed – PV power production)
T_{abs}	- absorber plate temperature
T_{amb}	- ambient temperature for convective losses from the top surface
T_{back}	- environment temperature for convective losses from the bottom surface
T_{fluid}	- bulk temperature of the fluid flowing through the solar collector
$T_{fluid,in}$	- temperature of the fluid flowing into the solar collector
$T_{fluid,out}$	- temperature of the fluid flowing out of the solar collector
T_{fluid}	- local fluid temperature
T_{PV}	- PV cell temperature

T_{sky} - sky temperature for long-wave radiation calculations
$\overline{T}$ - mean temperature
W - the width (x-direction) between adjacent fluid tubes in the collector
Width - the width of the collector
$X_{Cell\ Temp}$ - multiplier for the PV cell efficiency as a function of the cell temperature
X_{NS} - multiplier to account for collectors connected in series (thermally)
$X_{Radiation}$ - multiplier for the PV cell efficiency as a function of the incident radiation
y - a variable indicating the direction of flow through the collector (y=L is the collector outlet)

Subscripts

b - beam radiation
d - diffuse radiation
g - ground
G - radiation
h - total horizontal
n - normal incidence
nominal - refers to the reference conditions
PV - photovoltaic
s - sky diffuse
t - total (beam + diffuse)

Mathematical Description:

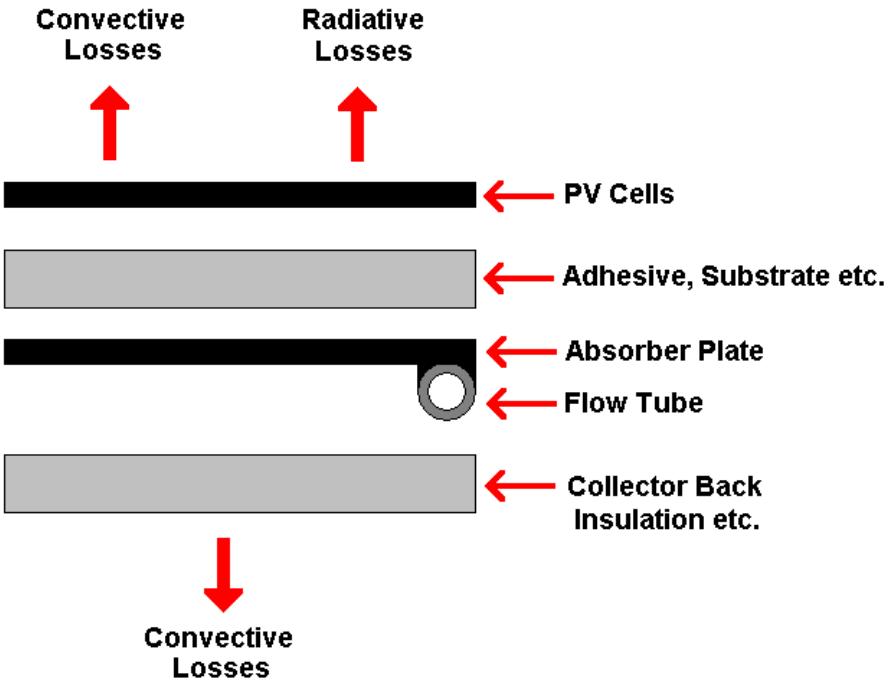


Figure 2 - PV/T Definitions

An energy balance on the collector surface (PV cells) at any point along the surface, (neglecting conduction along the surface) shows the following relationship:

$$0 = S - h_{outer}\left(T_{PV} - T_{amb}\right) - h_{rad}\left(T_{PV} - T_{sky}\right) - \frac{\left(T_{PV} - T_{abs}\right)}{R_T} \qquad \text{(Eq. 560.1)}$$

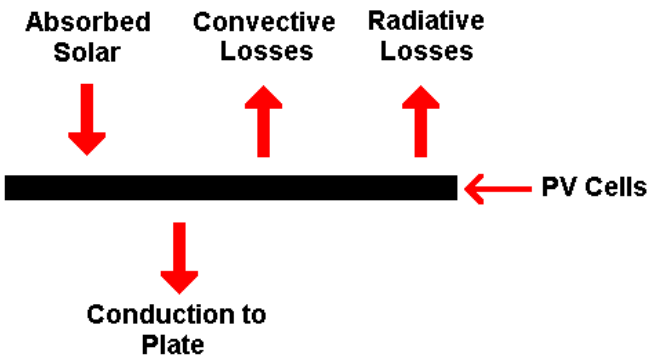


Figure 3: Surface Energy Balance

where:

$$R_T = R_1 \qquad \text{(Eq. 560.2)}$$

$$h_{rad} = \varepsilon\sigma\left(T_{PV} + T_{sky}\right)\left(T_{PV}^{\ 2} + T_{sky}^{\ 2}\right) \qquad \text{(Eq. 560.3)}$$

S is the net absorbed solar radiation and accounts for the absorbed solar radiation minus the PV power production. To account for off-normal solar radiation effects, the transmittance-absorptance product at normal incidence is multiplied by the following term in order to get the transmittance-absorptance at other incidence angles. This term is referred to as the incidence angle modifier (IAM).

$$IAM = \frac{(\tau\alpha)}{(\tau\alpha)_n} = \frac{G_{bT}\,\frac{(\tau\alpha)_b}{(\tau\alpha)_n} + G_d\,\frac{(1+\cos\beta)}{2}\,\frac{(\tau\alpha)_s}{(\tau\alpha)_n} + G_h\,\rho_g\,\frac{(1-\cos\beta)}{2}\,\frac{(\tau\alpha)_g}{(\tau\alpha)_n}}{G_T} \qquad \text{(Eq. 560.4)}$$

where:

$$\frac{(\tau\alpha)_b}{(\tau\alpha)_n} = 1 - b_0\left(\frac{1}{\cos\theta} - 1\right) \qquad \text{(Eq. 560.5)}$$

The incidence angle modifiers for both sky and diffuse radiation are determined by defining equivalent incidence angles for beam radiation that give the same transmittance as for diffuse radiation (Duffie and Beckman). The effective angles for sky diffuse and ground reflected radiation are:

$$\theta_{sky} = 59.68 \quad - \quad 0.1388\,\beta \quad + \quad 0.001497\,\beta^2 \qquad \text{(Eq. 560.6)}$$

$$\theta_{ground} = 90.0 \quad - \quad 0.5788\ \beta \quad + \quad 0.002693\ \beta^2 \qquad \text{(Eq. 560.7)}$$

With these definitions S, the net absorbed solar radiation, from equation 560.1 can be determined as:

$$S = (\tau\alpha)_n\ IAM\ G_T\ (1 - \eta_{PV}) \qquad \text{(Eq. 560.8)}$$

The efficiency of the PV cells is a function of the cell temperature and the incident solar radiation:

$$\eta_{PV} = \eta_{nominal}\ X_{CellTemp}\ X_{Radiation} \qquad \text{(Eq. 560.9)}$$

where:

$$X_{CellTemp} = 1 + Eff_T\ (T_{PV} - T_{ref}) \qquad \text{(Eq. 560.10)}$$

$$X_{Radiation} = 1 + Eff_G\ (G_T - G_{ref}) \qquad \text{(Eq. 560.11)}$$

An energy balance taken for a differential sized section along the absorber plate, at any point along the plate away from the tube section, shows the following relationship (assuming the plate is thin and made from a conductive material):

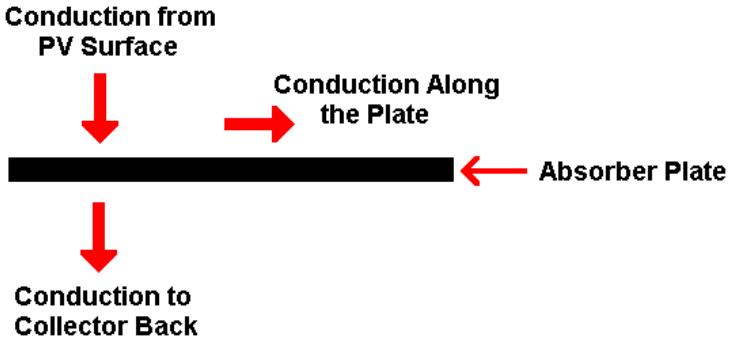


Figure 4: Absorber Fin Heat Flows

$$k\,\lambda \frac{d^2 T_{abs}}{dx^2} = \frac{(T_{abs} - T_{back})}{R_B} - \frac{(T_{PV} - T_{abs})}{R_T} \qquad \text{(Eq. 560.12)}$$

where:

$$R_B = R_2 + \frac{1}{h_{inner}} \qquad \text{(Eq. 560.13)}$$

This is a classical fin problem where the absorber plate section between the midpoint of two adjacent tubes and the tube acts as the fin. Solving equation 560.1 for T_{PV} and substituting into equation 560.12, we derive the following differential equation for the temperature distribution (x-direction) along the absorber plate:

$$\frac{d^2T_{abs}}{dx^2} = \frac{F'}{k\lambda}\left(T_{abs}\left(\frac{1}{R_TF'}+\frac{1}{R_BF'}-\frac{1}{R_T}\right)-\left(S+h_{rad}T_{sky}+h_{outer}T_{amb}+\frac{T_{back}}{R_BF'}\right)\right) \quad \text{(Eq. 560.14)}$$

where:

$$F' = \frac{1}{h_{rad}R_T + h_{outer}R_T + 1} \quad \text{(Eq. 560.15)}$$

We can recast equation 560.14 as:

$$\frac{d^2\Psi}{dx^2} - m^2\Psi = 0 \quad \text{(Eq. 560.16)}$$

where:

$$\Psi = T_{abs} - \frac{S+h_{rad}T_{sky}+h_{outer}T_{amb}+\frac{T_{back}}{R_BF'}}{\frac{1}{R_TF'}+\frac{1}{R_BF'}-\frac{1}{R_T}} \quad \text{(Eq. 560.17)}$$

$$m = \sqrt{\frac{F'\left(\frac{1}{R_TF'}+\frac{1}{R_BF'}-\frac{1}{R_T}\right)}{k\lambda}} \quad \text{(Eq. 560.18}$$

Solving equation 560.16 we find:

$$\Psi = C_1\sinh(mx) + C_2\cosh(mx) \quad \text{(Eq. 560.19)}$$

Equation 560.19 defines the temperature distribution along the plate in the x-direction, where x=0 is the mid-point between two adjacent tubes and x=(W-D_{tube})/2 is the base of the fin. To find the constants C_1 and C_2, we need to apply our boundary conditions. For this problem we have the boundary conditions from symmetry at the midpoint between adjacent tubes (x=0) and from the known base temperature (T_b) at x=(W-D_{tube})/2:

$$\frac{d\Psi}{dx} = 0 \quad at\ x = 0 \quad \text{(Eq. 560.20)}$$

$$\Psi = T_b - \frac{S + h_{rad}T_{sky} + h_{outer}T_{amb} + \frac{T_{back}}{R_B F'}}{\frac{1}{R_T F'} + \frac{1}{R_B F'} - \frac{1}{R_T}} \qquad \text{at } x = (W - D_{tube})/2 \qquad \text{(Eq. 560.21)}$$

Applying our boundary conditions and solving for C_1 and C_2 we find:

$$C_1 = 0 \qquad \text{(Eq. 560.22)}$$

$$C_2 = \frac{T_b - \left(\frac{S + h_{rad}T_{sky} + h_{outer}T_{amb} + \frac{T_{back}}{R_B F'}}{\frac{1}{R_T F'} + \frac{1}{R_B F'} - \frac{1}{R_T}} \right)}{\cosh\left(m \frac{(W - D_{tube})}{2} \right)} \qquad \text{(Eq. 560.23)}$$

Substituting C1 and C2 into equation 560.19, and then applying equation 560.17, we derive the expression for the temperature distribution along the plate as a function of the base temperature:

$$T_{abs}(x) = \frac{b}{j} + \left(T_b - \frac{b}{j} \right) \frac{\cosh(m\,x)}{\cosh\left(m \frac{(W - D_{tube})}{2} \right)} \qquad \text{(Eq. 560.24)}$$

where:

$$\frac{b}{j} = \left(\frac{S + h_{rad}T_{sky} + h_{outer}T_{amb} + \frac{T_{back}}{R_B F'}}{\frac{1}{R_T F'} + \frac{1}{R_B F'} - \frac{1}{R_T}} \right) \qquad \text{(Eq. 560.25)}$$

With the temperature distribution known along the fin (equation 560.24), we can calculate the energy conducted to the base from the fin:

$$q'_{fin} = -k\,\lambda \frac{dT_{abs}(x)}{dx} = k\,\lambda\,m\,(\frac{b}{j} - T_b) \tanh\left(m \left(\frac{W - D_{tube}}{2} \right) \right) \qquad \text{(Eq. 560.26)}$$

An energy balance on the base (non-fin) area of the absorber plate shows:

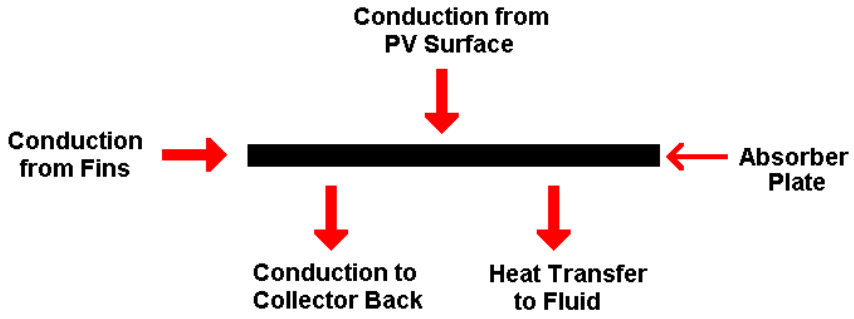


Figure 5: Heat Flows for Fin Base

$$q'_{fluid} = D_{tube}\left(\frac{T_{PV} - T_B}{R_T}\right) - D_{tube}\left(\frac{T_B - T_{Back}}{R_B}\right) + 2\,q'_{fin} \qquad \text{(Eq. 560.27)}$$

The useful energy gain to the fluid may also be expressed as a function of the base temperature:

$$q'_{fluid} = \left(\frac{T_B - T_{fluid}}{\frac{1}{h_{fluid}\,\pi\,D_{tube}} + \frac{1}{C_B}}\right) \qquad \text{(Eq. 560.28)}$$

An expression for the collector useful energy gain as a function of the fluid temperature may be derived by substituting terms from equations 560.1, 560.26 and 560.28 into equation 560.27 and re-arranging:

$$q'_{fluid} = \frac{\kappa}{\theta} T_{fluid} + \frac{\varepsilon}{\theta} \qquad \text{(Eq. 560.29)}$$

where:

$$\kappa = -D_{tube} F'\left(h_{rad} + h_{outer} + \frac{1}{R_B\,F'}\right) - 2\,k\,\lambda\,m\tanh\left(m\left(\frac{W - D_{tube}}{2}\right)\right) \qquad \text{(Eq. 560.30)}$$

$$\theta = 1 + D_{tube} F'\left(\frac{1}{h_{fluid}\,\pi\,D_{tube}} + \frac{1}{C_B}\right)\left(h_{rad} + h_{outer} + \frac{1}{R_B\,F'}\right) + 2\,k\,\lambda\,m\tanh\left(m\left(\frac{W - D_{tube}}{2}\right)\right)\left(\frac{1}{h_{fluid}\,\pi\,D_{tube}} + \frac{1}{C_B}\right) \qquad \text{(Eq. 560.31)}$$

$$\varepsilon = D_{tube} F' \left(S + h_{rad} T_{sky} + h_{outer} T_{amb} + \frac{T_{back}}{R_B F'} \right) + 2\,k\,\lambda\,m \tanh\left(m \left(\frac{W - D_{tube}}{2} \right) \right) \left(\frac{S + h_{rad} T_{sky} + h_{outer} T_{amb} + \frac{T_{back}}{R_B F'}}{\frac{1}{R_T F'} + \frac{1}{R_B F'} - \frac{1}{R_T}} \right) \quad \text{(Eq. 560.32)}$$

An energy balance taken around a differential section of fluid moving through the collector (in the y-direction) can be written as:

$$\dot{m} C_p \frac{dT_{fluid}}{dy} - N_{tubes}\ q'_{fluid} = 0 \quad \text{(Eq. 560.33)}$$

Subbing equation 560.29 into equation 560.33 we find:

$$\frac{dT_{fluid}}{dy} = \frac{N_{tubes}}{\dot{m} C_p} \frac{\kappa}{\theta} T_{fluid} + \frac{N_{tubes}}{\dot{m} C_p} \frac{\varepsilon}{\theta} \quad \text{(Eq. 560.34)}$$

Integrating this equation from zero to y we find:

$$T_{fluid}(y) = \left(T_{fluid,in} + \frac{\varepsilon}{\kappa} \right) \exp\left(\frac{N_{tubes}}{\dot{m} C_p} \frac{\kappa}{\theta} y \right) - \frac{\varepsilon}{\kappa} \quad \text{(Eq. 560.35)}$$

If we let y=L, we can solve for the fluid outlet temperature:

$$T_{fluid,out} = \left(T_{fluid,in} + \frac{\varepsilon}{\kappa} \right) \exp\left(\frac{N_{tubes}}{\dot{m} C_p} \frac{\kappa}{\theta} L \right) - \frac{\varepsilon}{\kappa} \quad \text{(Eq. 560.36)}$$

The collector useful energy gain can now be calculated:

$$Q_u = \dot{m} C_p \left(T_{fluid,out} - T_{fluid,in} \right) \quad \text{(Eq. 560.37)}$$

And the collector useful energy gain per unit length can be calculated as:

$$q'_u = q'_{fluid} = \frac{\dot{m} C_p \left(T_{fluid,out} - T_{fluid,in} \right)}{L\, N_{tubes}} \quad \text{(Eq. 560.38)}$$

The mean fluid temperature can be found by integrating the fluid temperature with respect to y and dividing by the flow length:

$$\overline{T}_{fluid} = \frac{1}{L} \int_0^L T_{fluid}(y)\, dy \quad \text{(Eq. 560.39)}$$

Using equation 560.35 and 560.39 and solving the differential equation we find:

$$\overline{T}_{fluid} = \left(\frac{T_{fluid,in} + \frac{\varepsilon}{\kappa}}{\frac{N_{tubes}}{\dot{m} C_p} \frac{\kappa}{\theta} L} \right) \exp\left(\frac{N_{tubes}}{\dot{m} C_p} \frac{\kappa}{\theta} L \right) - \left(\frac{T_{fluid,in} + \frac{\varepsilon}{\kappa}}{\frac{N_{tubes}}{\dot{m} C_p} \frac{\kappa}{\theta} L} \right) - \frac{\varepsilon}{\kappa} \qquad \text{(Eq. 560.40)}$$

With mean fluid temperature found from equation 560.40, and the collector useful energy gain per unit length found from equation 560.38, the mean base temperature can be solved from equation 560.28. With the mean base temperature solved, the temperature distribution across the absorber (fin section) can be found from applying equation 560.24.

The mean fin temperature can then be found by integrating the fin temperature function over the width of the fin, and dividing by the fin width:

$$\overline{T}_{fin} = \int_0^{\left(\frac{W - D_{tube}}{2}\right)} T(x) dx \qquad \text{(Eq. 560.41)}$$

$$\overline{T}_{fin} = \frac{S + h_{rad} T_{sky} + h_{outer} T_{amb} + \frac{T_{back}}{R_B F'}}{\frac{1}{R_T F'} + \frac{1}{R_B F'} - \frac{1}{R_T}} + \frac{\left(\overline{T}_B - \frac{S + h_{rad} T_{sky} + h_{outer} T_{amb} + \frac{T_{back}}{R_B F'}}{\frac{1}{R_T F'} + \frac{1}{R_B F'} - \frac{1}{R_T}} \right) \tanh\left(m \left(\frac{W - D_{tube}}{2} \right) \right)}{m \left(\frac{W - D_{tube}}{2} \right)} \qquad \text{(Eq. 560.42)}$$

The mean absorber temperature can then be found by area weighting the mean base temperature and the mean fin temperature:

$$\overline{T}_{abs} = \frac{\left(D_{tube} \overline{T}_B + \left(W - D_{tube} \right) \overline{T}_{fin} \right)}{W} \qquad \text{(Eq. 560.43)}$$

The mean PV surface temperature ($\overline{T}_{PV}$) can then be found from equation 560.1. The solution of this set of equations requires an iterative approach as S is a function of the mean PV surface temperature:

1. Guess a value for the PV surface temperature.
2. Calculate the radiation heat transfer coefficient using equation 560.3.
3. Calculate the PV efficiency using equations 560.9 and 560.10.
4. Calculate the net absorbed solar radiation using equation 560.8.

5. Calculate the fluid outlet temperature using equation 560.36 and the mean fluid temperature using equation 560.40.
6. Calculate the collector useful energy gain per unit length using equation 560.38.
7. Calculate the mean base temperature from Equation 560.28.
8. Calculate the mean fin temperature from Equation 560.42.
9. Calculate the mean absorber temperature from Equation 560.43.
10. Calculate the mean PV surface temperature using Equation 560.1 and repeat steps 2 to 9 until convergence is reached

With convergence attained, equation 6.9.3 from Duffie and Beckman can be used to find the overall loss coefficient from the collector (U_L):

$$Q_u = Area\left[S - U_L\left(\overline{T}_{abs} - T_{amb}\right)\right] \quad \text{(Eq. 560.44)}$$

Finally, with the collector overall loss coefficient calculated, the collector heat removal factor can be calculated from equation 6.7.6 of Duffie and Beckman:

$$Q_u = Area\ F_R\left[S - U_L\left(T_{fluid,in} - T_{amb}\right)\right] \quad \text{(Eq. 560.45)}$$

With the PV cell temperature converged the PV power can be calculated:

$$Power = \left(\tau\alpha\right)_n\ IAM\ G_T\ Area\ \ \eta_{PV} \quad \text{(Eq. 560.46)}$$

The remaining relevant heat transfers for the collector are then calculated as:

$$Q_{loss,top,conv} = h_{outer}\ Area\left(\overline{T}_{PV} - T_{amb}\right) \quad \text{(Eq. 560.47)}$$

$$Q_{loss,top,rad} = h_{rad}\ Area\left(\overline{T}_{PV} - T_{sky}\right) \quad \text{(Eq. 560.48)}$$

$$Q_{loss,back} = Area\frac{\left(\overline{T}_{abs} - T_{back}\right)}{R_B} \quad \text{(Eq. 560.49)}$$

$$Q_{PV\to Plate} = Area\left(\frac{\overline{T}_{PV} - \overline{T}_{ABS}}{R_T}\right) \quad \text{(Eq. 560.50)}$$

$$Q_{absorbed} = A\left(\tau\alpha\right)_n\ IAM\ G_T\left(1 - \eta_{PV}\right) \quad \text{(Eq. 560.51)}$$

An energy balance on the collector surface is then:

$$Q_{absorbed} = Q_{loss,top,conv} + Q_{loss,top,rad} + Q_{PV\to plate} \quad \text{(Eq. 560.52)}$$

An energy balance on the entire collector can also be written:

$$Q_{absorbed} = Q_{loss,top,conv} + Q_{loss,top,rad} + Q_u + Q_{loss,back} \quad \text{(Eq. 560.53)}$$